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Quantum model of emission in a weakly non ideal plasma

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Abstract. We present, in this work a simple analytical expression of line emission in weakly non ideal plasma using a simplified quantum model. This formalism allows to explain the variations of the line widths with the density in a weakly non ideal plasma. We apply this model to plasma neutral helium lines HeI 6678 Å and HeI 5876 Å and explain the non linearity of the line width.

PACS. 32.70.Jz Line shapes, widths, and shifts – 31.15.Lc Quasiparticle methods – 42.50.-p Quantum optics

1 Introduction

Stark broadening of spectral lines is important for physical diagnostics and astrophysical modelling. In fact, it is found to be a reliable tool for understanding the characteristics of the plasma. This requires, in practice, a detailed knowledge of line profile in the plasma. Collisions between ions and electrons play an important role in Astrophysics for the interpretation of line spectra and for the modelling of the stellar interiors.

The diagnostic of a plasma rest on the knowledge of the spectral profile. For a classic plasma, in a thermodynamic equilibrium, where one supposes that collisions between emitters and rapid particles of the plasma gives a Lorentzian profile [1–6]. Slow charged particles, namely ions, contribute by a quasistatic field: the ionic microfield, whose distribution is presented in several works [7–10]. When the density of the plasma increase, N-particles interactions effects appears; the middle becomes dense and the state of the plasma become non ideal identified by the non ideality factor Γ.

This non-ideality factor Γ is defined as the ratio of the mean interaction potential energy between charged particles U to their kinetic energy E_c ($\Gamma = U/E_c = 2.26 \times 10^{-5} N_e^{1/3}/T$) where N_e is in m^{-3} and T in K.

In the range of $0.001 \lesssim \Gamma \leq 0.05$ the plasma is very weakly non-ideal, for $0.05 \leq \Gamma \leq 0.25$ the plasma is weakly non-ideal. When Γ varied between 0.25 and 0.5 the plasma is non-ideal and for $\Gamma > 0.5$, the plasma in strongly non-ideal.

Plasma shielding effects due to electron and ion correlations are not negligible in the physical conditions of white dwarfs atmospheres, owing to their high density. They also play a role in the case of rather cool stars and for atomic transitions which are quasi-degenerated. Electronic correlations (screening effects) are usually taken into account by introducing a cutoff in the interaction when the electron-atom distance exceeds the Debye radius R_D. A more consistent treatment to describe collective effects is the Debye-Hückel potential: the two-particles Coulomb field is shielded by the ensemble of the surrounding electrons. This is a good approximation only for high temperature and low density plasmas (weakly non ideal plasmas). Several works study proprieties of non-ideal plasma using semiclassical formalism with the Coulomb cutoff potential [11,12] or with the ion sphere potential [13]. These potentials, which can be written as the Coulomb potential with two correcting terms, are widely used in the literature [14,15].

In this paper, we investigate full quantum model based on quasiparticles treatment to describe the electron ion interaction in a non ideal plasma. The main aim of this work is to present an expression of the spectral line profile when the plasma is non ideal. We developed a quantum formalism of the emission which take into account the interaction between particles such that it becomes applicable to a weakly non-ideal plasma. We give analytic expression of the line width and explain the non linearity of the width via the density in three experiments:

– experiment of Gauthier et al. [16] where Γ is between 0.081 and 0.096;
– experiment of El Bezzari [17] where Γ is close to 0.1, theses two experimental measurements reports Stark broadening of the HeI 2^1P-3^1D transition at 6678 Å;
– experiment of Büscher et al. [18] which report measurements of Stark broadening of the HeI 2^3P-3^3D transition at 5876 Å, where the non ideality factor is $0.037 \leq \Gamma \leq 0.045$.

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2 Presentation of the model

We consider a plasma with ions and electrons in interactions. This emitting system is equivalent to *quasiparticles* formed by ions and electrons in interaction with photons. The quasiparticles model is applied in many fields of physics as in semiconductors [19–21], in Bose-Einstein condensation [22] and in the microfield distributions functions [9,10].

The Hamiltonian of the system can be written in the form:

$$H = H_{qp} + H_F + H_{int} + H_{nl} \tag{1}$$

where

$$H_{qp} = \hbar\omega_{qp}\hat{b}^+\hat{b} \tag{2}$$

with $\hat{b}^+$ and $\hat{b}$ the bosonic creation and annihilation operators of a quasiparticle, verifying the relation of commutation $[\hat{b}, \hat{b}^+] = 1$

$$H_F = \hbar\omega_F\hat{a}^+\hat{a}. \tag{3}$$

H_F is the Hamiltonian of the photonic field, $\hat{a}^+$ and $\hat{a}$ are the creation and annihilation operators of a photon verifying the commutation relation $[\hat{a}, \hat{a}^+] = 1$.

The Hamiltonian that describes the interaction between a quasiparticle and a photon can be written as

$$H_{int} = \hbar g\left(\hat{a}^+\hat{b} + \hat{b}^+\hat{a}\right) \tag{4}$$

where g is the coupling constant.

ω_{qp} et ω_F are respectively proper frequencies of the quasiparticle and the photon that we supposes in resonance ($\omega_{qp} = \omega_F = \omega_0$).

The interaction Hamiltonian takes into account two process that are:

- annihilation of a photon and creation of a quasiparticle;
- annihilation of a quasiparticle and a creation of a photon.

The non linear Hamiltonian which describes the interaction between quasiparticles is:

$$H_{nl} = \alpha\hbar\hat{b}^+\hat{b}^+\hat{b}\hat{b} \tag{5}$$

where α is the interaction constant.

We take into account the interaction with the thermal bath through a dissipative term $\gamma/2$.

3 Line profile

The profile of a line in a plasma is written as:

$$\hat{I}(\omega) = TF\left\{\hat{C}(t)\right\} = \int_{-\infty}^{\infty}\hat{C}(t)e^{-i\omega t}dt \tag{6}$$

where

$$\hat{C}(t) = \left\langle\hat{a}^+(0)\hat{a}(t)\right\rangle \tag{7}$$

is the correlation function. In interaction representation, we take:

$$\hat{a} = \bar{a}e^{-i\omega_0 t}, \qquad \widehat{b} = be^{-i\omega_0 t}. \tag{8}$$

We transform the $\bar{a}$ and $\bar{a}^+$ operators respectively into

$$a = i\bar{a}, \qquad a^+ = -i\bar{a}^+. \tag{9}$$

The commutation equation still: $[a, a^+] = 1$, where a^+ and a can be considered as creation and annihilation operators.

For simplifications we define

$$C(t) = \left\langle a^+(t)a(0)\right\rangle, \quad I(\omega) = TF\left\{C(t)\right\} \tag{10}$$

so

$$\hat{C}(t) = C(t)e^{i\omega_0 t}, \quad \hat{I}(\omega) = I(\omega - \omega_0). \tag{11}$$

In this case, the evolution operator $a(t)$ can be deduced from the evolution equation of the density operator [23–26]:

$$\begin{aligned}\frac{d\rho}{dt} &= \frac{1}{i\hbar}\left[H_I, \rho\right] + \frac{\gamma}{2}\left(2a\rho a^+ - a^+a\rho - \rho a^+a\right)\\ &\quad + \gamma n_{th}\left(a\rho a^+ + a^+\rho a - a^+a\rho - \rho a^+a\right)\end{aligned} \tag{12}$$

where

$$H_I = i\hbar g(a^+b - b^+a) + \hbar\alpha b^+b^+bb \tag{13}$$

and $\gamma/2$ takes into account all radiatives dissipations of the plasma. n_{th} is the average number of the thermal photons.

The Master equation, in interaction representation becomes:

$$\begin{aligned}\frac{d\rho}{dt} &= g\left[a^+b - b^+a, \rho\right] - i\alpha\left[b^+b^+bb, \rho\right]\\ &\quad + \frac{\gamma}{2}\left(2a\rho a^+ - a^+a\rho - \rho a^+a\right)\\ &\quad + \gamma n_{th}\left(a\rho a^+ + a^+\rho a - a^+a\rho - \rho a^+a\right).\end{aligned} \tag{14}$$

By writing that $\langle a\rangle = \langle a(t)\rangle = \mathrm{tr}\left(a(t)\rho(0)\right)$ where $a(t) = U^+(t,0)a(0)U(t,0)$ and $\rho(t) = U(t,0)\rho(0)U^+(t,0)$ and similarly for b; $U(t,0)$ represents the evolution operator. We obtains then evolution equations for mean values a and b in the interaction representation:

$$\begin{cases}\dfrac{d}{dt}\langle a\rangle = g\langle b\rangle - \frac{\gamma}{2}\langle a\rangle\\ \dfrac{d}{dt}\langle b\rangle = -g\langle a\rangle - 2i\alpha N\langle b\rangle\end{cases} \tag{15}$$

where we used $\langle b^+bb\rangle \approx \langle b^+b\rangle\langle b\rangle = N\langle b\rangle$, N is the total quasiparticle number.

The system has input fluctuations F^{in} in the quasiparticle mode due to interactions with external environment.

In this case, evolution equations of operators a and b have the same forms as the preceding equations but with input fluctuations

$$\begin{cases}\dfrac{d}{dt}b = -ga - 2i\alpha Nb\\ \dfrac{d}{dt}a = gb - \frac{\gamma}{2}a + \sqrt{\gamma}F^{int}\end{cases}. \tag{16}$$

This last equation can be considered as a Langevin equation where input fluctuations $\sqrt{\gamma}F^{in}$ are Langevin field associated to the photons thermal bath.

F^{in} represent fluctuations of the thermal bath [27,28]:

$$\begin{aligned}\left\langle F^{in}(t)\right\rangle &= 0\\ \left\langle F^{in}(t)F^{in}(t')\right\rangle &= 0\\ \left\langle F^{in+}(t)F^{in}(t')\right\rangle &= n_{th}\delta(t-t')\\ \left\langle F^{in}(t)F^{in+}(t')\right\rangle &= (n_{th}+1)\delta(t-t').\end{aligned} \tag{17}$$

In the Fourier space, fluctuations verify:

$$\begin{aligned}\left\langle F^{in}[\omega]\,F^{in}[\omega']\right\rangle &= 0\\ \left\langle F^{in+}[\omega]\,F^{in}[\omega']\right\rangle &= 2\pi n_{th}\delta(\omega+\omega')\\ \left\langle F^{in}[\omega]\,F^{in+}[\omega']\right\rangle &= 2\pi(n_{th}+1)\delta(\omega+\omega').\end{aligned} \tag{18}$$

The Fourier transform of evolution equations of operators a and b are:

$$\begin{cases} i\omega a[\omega] = gb[\omega] - \dfrac{\gamma}{2}a[\omega] + \sqrt{\gamma}F^{in}[\omega] \\ i\omega b[\omega] = -ga[\omega] - 2i\alpha N b[\omega] \end{cases}. \tag{19}$$

The origin of the spectral frequency used is the transition frequency ω_0.

The evolution matrix defined by:

$$M[\omega]\begin{bmatrix} a[\omega] \\ b[\omega]\end{bmatrix} = \begin{bmatrix}\sqrt{\gamma}F^{in}[\omega] \\ 0\end{bmatrix} \tag{20}$$

is then:

$$M[\omega] = \begin{pmatrix}\gamma/2 + i\omega & -g \\ g & i\omega + 2i\alpha N\end{pmatrix}. \tag{21}$$

Resolution of the system gives

$$a[\omega] = \sqrt{\gamma}F^{in}[\omega]\,T[\omega] \tag{22}$$

where

$$T[\omega] = \frac{i(\omega+2\alpha N)}{g^2 + i(\omega+2\alpha N)(\gamma/2+i\omega)} \tag{23}$$

and we find that:

$$a^+[\omega] = \sqrt{\gamma}F^{in+}[\omega]\,T^+[\omega] \tag{24}$$

so

$$\left\langle a^+[\omega']\,a[\omega]\right\rangle = \gamma T^+[\omega']\,T[\omega]\left\langle F^{in+}[\omega']\,F^{in}[\omega]\right\rangle \tag{25}$$

or

$$\left\langle a^+[\omega']\,a[\omega]\right\rangle = 2\pi n_{th}\gamma T^+[\omega']\,T[\omega]\,\delta(\omega+\omega').$$

On the other hand, we obtain from equation (10):

$$\left\langle a^+[\omega']\,a[\omega]\right\rangle = 2\pi I[\omega]\,\delta(\omega+\omega') \tag{26}$$

so that

$$n_{th}\gamma T^+[\omega']\,T[\omega]\,\delta(\omega+\omega') = I[\omega]\,\delta(\omega+\omega'). \tag{27}$$

The integration of this last equation over ω' in the interval $[\omega-\delta\omega, \omega+\delta\omega]$ and the relation $T^+[\omega] = \left(T[-\omega]^*\right)$ gives

$$I[\omega] = \gamma n_{th}\left|T[\omega]\right|^2 \tag{28}$$

so

$$I[\omega] = \frac{\gamma n_{th}(\omega+2\alpha N)^2}{\left(g^2-\omega^2-2\alpha N\omega\right)^2 + \left(\dfrac{\gamma}{2}\right)^2(\omega+2\alpha N)^2} \tag{29}$$

$$= \frac{\gamma n_{th}}{\left(\dfrac{g^2}{\omega+2\alpha N}-\omega\right)^2 + \left(\dfrac{\gamma}{2}\right)^2} \tag{30}$$

which gives as line profile:

$$\hat{I}[\omega] = \frac{\gamma n_{th}}{\left(\dfrac{g^2}{\Delta\omega+2\alpha N}-\Delta\omega\right)^2 + \left(\dfrac{\gamma}{2}\right)^2} \tag{31}$$

where $\Delta\omega = \omega - \omega_0$.

In absence of coupling ($g = 0$) and non linear interaction ($\alpha = 0$), we find the usual Lorentzian expression of the profile for ideal plasmas.

For weakly non ideal plasmas, the values of the coupling and the non linear interaction are weakly compared to the dissipation of the system

$$\left(g^2 \ll \left(\frac{\gamma}{2}\right)^2 \text{ and } (\alpha N)^2 \ll \left(\frac{\gamma}{2}\right)^2\right).$$

As we consider that the coupling between the photon and the emitter (g being the coupling constant) is very weak ahead the quasiparticle interactions αN which is self weak ahead the dissipations $\gamma/2$ of the emitter, the width $\gamma'/2$ of the spectral profile is $\gamma'/2 = \omega_2 - \omega_1$where ω_1 and ω_2 are solutions of the following equation:

$$\left(\frac{g^2}{\Delta\omega+2\alpha N}-\Delta\omega\right)^2 = \left(\frac{\gamma}{2}\right)^2 \tag{32}$$

so, we obtain the modified width γ':

$$\gamma' = \gamma + \frac{(4g)^2-(2\alpha N)^2}{\gamma}. \tag{33}$$

For the N perturbers, the full width at half maximum (FWHM) is then

$$w' = N\gamma' = \gamma_1 N - \gamma_2 N^3 \tag{34}$$

where

$$\gamma_1 = \gamma + \frac{16g^2}{\gamma} \tag{35}$$

and

$$\gamma_2 = \frac{4\alpha^2}{\gamma}. \tag{36}$$

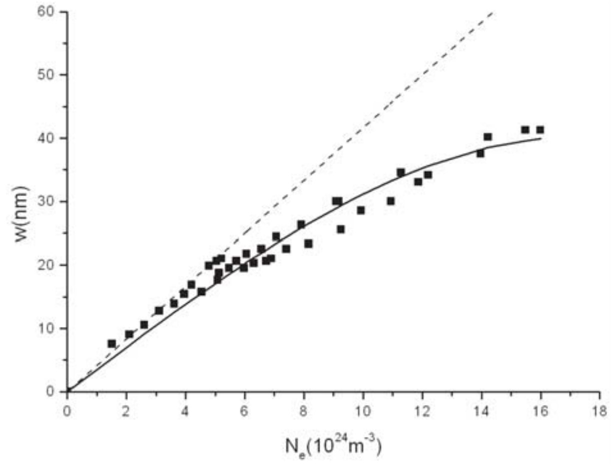

Fig. 1. Stark width versus the electron density for the helium line HeI 2^1P $-$ 3^1D transition at 6678 Å. Experimental data of Gauthier et al. [16]. The dashed line present the Griem theory [1]. The solid curve present our work.

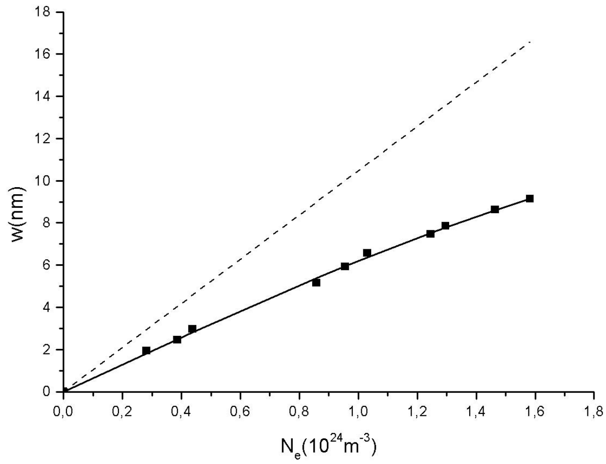

Fig. 2. Stark width versus the electron density for the helium line HeI 2^1P $-$ 3^1D transition at 6678 Å. Experimental data of Bezzari et al. [17]. The dashed line present the Griem theory [1]. The solid curve present our work.

In the expression (34) appear two terms. The first one (35) is proportional to the density N and it increases with the coupling constant. The second term (36) is proportional to N^3, it describes the non-linearity effect due to the interaction between quasiparticles, this negative term causes the decreasing of the widths for high density.

4 Applications to the neutral helium

We have applied our model to three experiments of helium lines and compared with Griem theory [1].

- Experiment of Gauthier et al. [16] which report measurements of Stark profiles of the HeI 2^1P-3^1D transition at 6678 Å (Fig. 1). Line shape have been measured at densities in the range of $N = 0.1$ to 2×10^{24} m^{-3} and temperature of 1.16 to 2.32×10^4 K which yield for the non ideality factor $0.081 \leq \Gamma \leq 0.096$.
- Experiment of El Bezzari [17] which report measurements of Stark profiles of the same 6678 Å line (Fig. 2). Line shape have been measured at densities in the range of $N = 0.3$ to 1.7×10^{24} m^{-3} and temperature of 2 to 3×10^4 K which yield for the non ideality factor Γ to be close to 0.1.
- Experiment of Büscher et al. [18] which report measurements of Stark profiles of the HeI 2^3P-3^3D transition at 5876 Å (Fig. 3). Line shape have been measured at densities in the range of $N = 0.5$ to 2.5×10^{24} m^{-3} and temperature of 4.64 to 6.38×10^4 K which yield for the non ideality factor $0.037 \leq \Gamma \leq 0.045$.

Our model gives a good fitting to experiments values. Table 1 gives the γ_1 and γ_2 values for the different experiments where N is in 10^{24} m^{-3} and w in nm.

The values of the non linear interaction constants are in the three experiments very small, which confirm our proposal about the weakly interactions of the quasiparticles in the plasma. Our model follows the experimental values better then the Griem linear model.

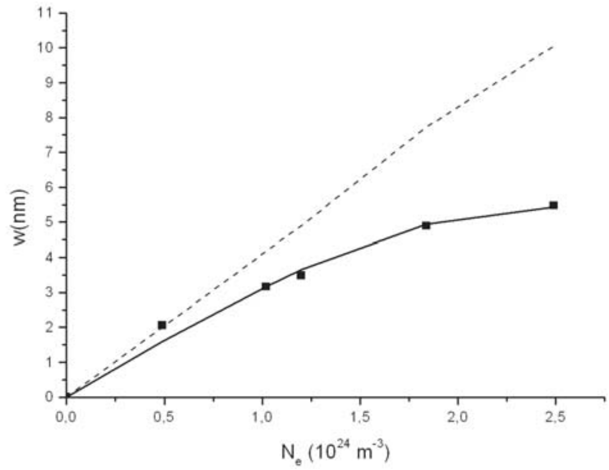

Fig. 3. Stark width versus the electron density for the helium line HeI 2^3P $-$ 3^3D transition at 5876 Å. Experimental data of Büscher et al. [18]. The dashed line present the Griem theory [1]. The solid curve present our work.

Table 1. γ_1 and γ_2 values for the different experiments where N is in 10^{24} m^{-3} and w in nm.

Experiment	γ_1	γ_2
Gauthier	3.51333	0.00399
El Bezzari	6.48037	0.28272
Büscher	3.29944	0.18039

5 Conclusion

In this work, we have elaborated a simplified quantum formalism of a line profile in a weakly non ideal plasma, that gives a Lorentzian profile with modified width, when interactions between constituents of the plasma are very weak.

This formalism allows to explain the variations of the line widths with the density in a weakly non ideal plasma where we find a modification of the Lorentzian profile. This modification is due to the interaction between quasi-particles in non ideal plasma. We have applied this model to three experiments where the authors found in the range of high density, the width of spectral lines grows nonlinearly.

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